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Additional Aerodynamic Features of Wing-Gurney Flap Flows

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An extension of the existing explanation of the flow mechanisms characteristic of the wing-Gurney flap flow has been proposed. Based on the Helmholtz vortex theory, it has been postulated at the onset of the present study that, in addition to the spanwise vortices, as predicted by a commonly accepted flow hypothesis one or more pairs of streamwise trailing vortices ought to exist. A series of wind-tunnel models, including a clean wing and one full-span and two partial-span Gurney flaps, have been tested in a low-speed wind tunnel. Several techniques, including two methods of flow visualization of trailing vortices and measurements of rotational speeds produced by these vortices, have been employed. All of these methods have consistently and clearly indicated the presence of additional vortices. As such, these vortices are important for accurate prediction of the aerodynamic characteristics of wing-Gurney flap configurations.

Nomenclature

A	=	cross-sectional area of a vortex tube, mm^2
$\text{Æ} = b^2/S$	=	wing aspect ratio
b	=	wing span, mm
C_D	=	wing drag coefficient
C_L	=	wing lift coefficient
c	=	wing chord, mm
h	=	height of Gurney flap, percent of wing chord
n	=	normal to surface
S	=	wing reference area, mm^2
s	=	wing semispan, mm
V_∞	=	freestream velocity, m/s
x, y, z	=	axes of wind coordinate system
α	=	angle of attack, deg
Γ	=	vortex circulation, m^2/s
ζ	=	vorticity, s^{-1}
ρ_∞	=	air density, slug/ft^3 kg/m^3
ω	=	angular velocity, s^{-1}

Introduction

GURNEY flap is a thin, relatively long and narrow plate positioned at the trailing edge of a wing, on its pressure side, perpendicular to the chord plane (for example, see Ref. 1). It has been used to increase the lift of the wing. The flow mechanism by which this is accomplished is discussed next, including both the commonly accepted hypothesis and an extension to it proposed by the present author. A somewhat detailed review of the relevant past research on the topic has also been included.

Gurney flap was first used on racecars by Dan Gurney in the 1970s when some of his cars having this modification achieved dramatic improvements through increased cornering speeds. These higher cornering speeds were attributed to the higher downforce, or the increased negative lift, generated by the inverted flap-equipped wings. In the area of aerospace applications, various Gurney flap implementations have been investigated involving rectangular, tapered, and delta planforms (see the following). However, it never found a prominent application in airplane design, and the only significant exemption that the author is aware of is its use on the McDonnell Douglas DC-10 and MD-11 airliners.

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Review of Past Research on Gurney Flaps

Since the late 1970s, many researchers have addressed the aerodynamics of Gurney flaps. Given next is a somewhat detailed review of the, in the author's opinion, most significant studies on Gurney flaps relevant to the present study, in a chronological order.

Liebeck[2] studied the effect of Gurney flaps on the performance of a wing having a Newman airfoil and found that a flap having a height equal to 1% of the wing chord increased the lift and decreased the drag of the wing. He also offered a hypothesis of the flow conditions about the Gurney flap, which is discussed next. Katz and Largman studied experimentally two-element wing configurations intended for racecar applications with and without Gurney flaps.[3] They found that the Gurney flap significantly increased the lift of the baseline airfoil, throughout a wide range of angles of attack. The maximum lift coefficient of the flapped wing increased too, whereas the lift-to-drag ratio decreased.[3]

Storms and Jang studied the performance changes of a rectangular wing caused by the addition of a Gurney flap and vortex generators, separately and combined.[4] They reported a significant lift augmentation for the configurations with Gurney flaps, increasing with the increased flap height. They also found that, at low-to-moderate lift coefficients, there was a drag penalty associated with the Gurney flap, whereas at higher lift coefficients the drag was significantly reduced. Ashby conducted an extensive study of the effects of Gurney flaps of varying heights and locations on the lift, drag, and pitching-moment characteristics of a two-element rectangular wing.[5] The lift-enhancing effectiveness of the flaps was found to be a function of the flap height, the airfoil element to which the flap was attached, and the second wing element deflection angle. Myose et al. investigated the effect of Gurney flaps having varying heights and spans on three-dimensional wings in a low-speed wind tunnel.[6] Straight and tapered three-dimensional wings with natural-laminar-flow (NLF) airfoil sections were tested. Compared to the 5-ft(1.524 m)-span clean wing, the 4.5-ft(1.372 m)-span, $0.017c$ and $0.033c$ height Gurney flaps increased the maximum lift coefficient by 17 and 22%, respectively. The increase in maximum lift coefficient was proportionately smaller with the shorter span Gurney flaps. With these flaps, changing the spanwise position from outboard to inboard resulted in a small improvement in performance. Improvements in the maximum lift coefficient were also obtained using Gurney flaps on a tapered NLF wing. The stall angle was decreased while the lift, drag, and nose-down pitching moment all increased. The lift-and-drag curve slopes were also changed.[6]

An experimental investigation of the effects of Gurney flaps on a reflection plane model was conducted by Heron et al.[7] Two sizes of Gurney flaps were tested on a series of configurations, which included a tapered wing, a fuselage, a nacelle, and their permutations. Results indicated that lift and drag were increased when using the Gurney flaps; lift-to-drag and lift-squared-to-drag ratios were also increased. Pitching moment became more negative with the Gurney flap. The efficiency of the full "airplane" with the small Gurney flap was close to that of the same configuration without the flap.[7] Giguère et al. tested a total of seven Gurney flaps of varying height in a wind tunnel on a truncated airfoil.[8] They showed that there was a flow-based scaling for the size of the optimum Gurney flap for best lift-to-drag performance and that a purely geometric criterion based on the chord of the airfoil might be misleading in determining the size of a beneficial Gurney flap. The boundary-layer thickness at the trailing edge on the pressure side of the baseline airfoil was shown to be a proper scaling parameter. For beneficial and optimal Gurney flap performance, it turned out to be most important that the flap be submerged within the local boundary layer.[8]

A two-dimensional numerical investigation was performed to determine the effect of a Gurney flap on a NACA 4412 airfoil.[9] The flowfield around the airfoil was predicted using an incompressible Navier–Stokes solver in combination with a one-equation turbulence model. Six Gurney flap sizes between 0.5 and 3.0% of the airfoil chord were studied. These computational results were compared with available experimental results. The numerical solutions showed that some Gurney flaps increased the airfoil lift coefficient with only a slight increase in drag coefficient. These solutions also showed the details of the flow structure at the trailing edge and thus provided a possible explanation for the improved aerodynamic performance.[9]

The effect of Gurney flaps on two-dimensional airfoils, three-dimensional wings, and a reflection plane model was investigated in a low-speed wind tunnel by Myose et al.[10] Compared to the baseline configuration, the Gurney flap improved the maximum lift coefficient. There was, however, a drag penalty associated with this increase in lift. On the three-dimensional wings, there was a slight improvement in performance when the Gurney flap was located inboard rather than outboard.[10] Van Dam et al.[11] studied the effects of Gurney-flap-like modifications for a three-dimensional wing with a similar section shapes as those used in the study reported in Ref. 10. They investigated the effects of both solid and serrated Gurney flaps. Their results were largely in agreement with those of Ref. 10 in terms of the overall effect of Gurney flaps on the lift characteristics of wings. They also found that the addition of Gurney flaps resulted in an increase of $(L/D)_{\max}$ of around 6%. As the possible major result of their study, they suggested that continuous Gurney flaps were not required in order to be effective on three-dimensional configurations.

A wind-tunnel investigation was undertaken to determine the effects of Gurney flaps on a 70-deg planar delta wing.[12] Both upper- and lower-surface leading-edge-, as well as trailing-edge, flaps were investigated. Results were presented including force balance, on- and off-surface flow visualization, as well as flowfield surveys. The data indicated that the lower surface leading-edge flaps increased the maximum lift coefficient and poststall lifting ability. The trailing-edge Gurney flap decreased the zero-lift angle of attack, thereby increasing lift for a given angle of attack, and also increased the maximum and poststall lift coefficient. Both of these configurations improved the wing efficiency at moderate-to-high lift coefficients. The devices did not greatly affect the longitudinal stability of the wing, although the trailing-edge flap generated an increase in nose-down pitching moment. The lower-surface leading-edge flap caused a moderate delay in the onset and progression of vortex breakdown over the wing. Upper-surface leading-edge flaps degraded performance.[12]

Jeffrey et al. studied the flow in the trailing-edge region of a single element wing fitted with Gurney flaps.[13] All of the Gurney flaps increased the lift at a given prestall α and also increased the drag at most values of C_L, leading to reductions in the maximum lift-to-drag ratio. Despite a reduction in stalling incidence, the Gurney flaps still increased $C_{L\max}$. Their data suggest a twin vortex structure downstream of the Gurney flap and that the wake consists of a von Kármán vortex street of alternately shed vortices, and they confirmed this flow structure by smoke visualization of the flow downstream of the Gurney flap. They suggested that the vortex shedding increases the trailing-edge suction of the airfoil, whereas the upstream face of the device decelerates the flow at the trailing edge of the pressure surface. These two changes result in a pressure difference acting across the trailing edge, and it is this that generates the increase in circulation and thus lift.[13] Buchholtz and Tso studied the effects of leading-edge fences and Gurney flaps, separately and combined, on the performance of a 60-deg delta wing to determine their lift augmentation effects for low-speed and low-angle-of-attack applications.[14] The primary effect of the Gurney flap was found to be an improvement of the circulation at the trailing edge. They found that the lift coefficient increased with increasing flap chord and that the increases amounted to 4- to 10-deg shifts of the lift curves. They also found that the Gurney flap increased the L/D ratio at higher lift coefficients and lowered the angle of attack at which vortex bursting reaches the trailing edge. Furthermore, the addition of Gurney flap produced significant nose-down pitching moment.

Jeffrey et al. studied various aspects of the aerodynamics of Gurney flaps fitted to a double-element wing.[15] Two typical flap angles were included. Force and pressure measurements and flow surveys using laser Doppler anemometry were conducted. Their measurements suggested that the wake flow behind the Gurney flap consists of a von Kármán vortex street of alternately shed vortices (see preceding comments on Ref. 13). The overall effects of a Gurney flap were found to be broadly similar whether it is fitted to a single-element or to the flap of a double-element wing. The overall increases in circulation were found to be lower than found for single-element wings, and they decreased with flap angle.[15] Li and Wang tested delta wings with and without riblets and Gurney flaps.[16] They found that, in comparison with the baseline clean configuration, all Gurney flaps increased the lift coefficient; the larger the Gurney flap the more pronounced this effect was. They also reported a shift of the lift curve upward and slightly to the left for a flapped configuration. These lift enhancements, however, were accompanied with increases in drag, such that at lower lift coefficient the clean wing produced higher L/D values, whereas at higher lift coefficients the configuration employing the Gurney flap, having a height equal to 1% of the wing chord, proved superior to the clean wing.

Gai and Palfrey conducted wind-tunnel experiments on a wing equipped with $0.05c$ Gurney flaps, solid and triangularly serrated.[17] Their results showed a dramatic increase in lift. Also, the angle of stall with the flaps attached was reduced. The serrated flap showed slightly lower drag than the solid one. The $(L/D)_{\max}$ for both flapped configurations was about 7% less than the corresponding baseline value. The configurations with Gurney flaps employed exhibited significant nose-down pitching moment about the quarter-chord. Based on their experiment and some previously reported data on Gurney flaps, a simple analysis, relating the effectiveness of the Gurney flap in enhancing lift to the flap parameters, was given.[17] Li et al.[18] tested a rectangular wing having a symmetrical airfoil and equipped with a Gurney flap. They varied the angle between the wing's lower surface and the Gurney flap, the mounting angle, and the distance between the flap and the wing trailing edge, the mounting location. They found that the Gurney flap at all tested mounting angles increased lift; the highest $C_{L,\max}$ being at 90 deg. This improved lift was accompanied with higher drag. When shifted forward from the trailing edge, the Gurney flap caused a decrease in lift and an increase in drag.

Wing-Gurney Flap Flow Mechanism: Present Hypothesis

The flow around the trailing edge of a wing equipped with a Gurney flap is depicted in Fig. 1, reproduced here from Ref. 2. Liebeck reported that wind-tunnel tests showed a significant turning of the flow over the backside of the flap.[2] A tufted probe also indicated a region of reverse flow behind the flap, which he modeled by two vortices as shown in Fig. 1. According to him, the wake deficit for the flapped configuration could be less than that in the case of a clean wing, particularly for thick airfoils, and he attributed the measured drag decrease to this effect. Although Liebeck did not specifically state that the added clockwise rotating vortex in the case of a Gurney flap is what increases the wing circulation and thus its lift, this appears to be a logical deduction to make.

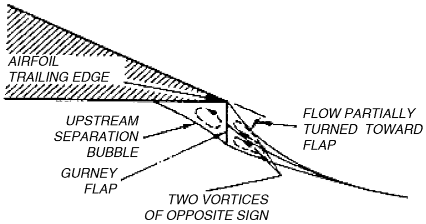

Fig. 1 Hypothesized trailing-edge flow conditions of an airfoil with a Gurney flap after Liebeck.[2]

Jeffrey et al.[13] produced experimental results, which matched closely the flow situation behind a Gurney flap as hypothesized by Liebeck. Gai and Palfrey[17] performed a simple analysis of the Gurney flap flow based on the flow hypothesis of Liebeck. They point out that the pressure difference at the trailing edge, because of the presence of the flap, causes the streamlines in that region to become highly curved. As already mentioned, the numerical analysis performed by Jang et al.[9] showed some details of the flow structure at the trailing edge and provided a possible explanation for the improved aerodynamic performance of the flapped wing. The computed flowfield in the vicinity of the flapped-wing trailing edge indicated a recirculation region in front of the flap. A strong clockwise-rotating vortex was apparent on the upper backside of the flap, which is in agreement with Liebeck's hypothesis. No discernable counter-rotating vortex could be seen on the lower portion of the flap backside, which was predicted by Liebeck's explanation. No other vortex structures, spanwise or streamwise, have been found.

As has been mentioned earlier, the measurements performed by Jeffrey et al.[13,15] indicated that the wake flow behind the Gurney flap consists of a von Kármán vortex street of alternately shed vortices. Their data suggest a twin vortex structure downstream of the Gurney flap and that the wake consists of a von Kármán vortex street of spanwise vortices, and they confirmed this flow structure by smoke visualization of the flow downstream of the Gurney flap. They suggested that the vortex shedding increases the trailing-edge suction of the airfoil, whereas the upstream face of the flap decelerates the flow at the trailing edge of the pressure surface. These two changes result in a pressure difference acting across the trailing edge, and it is this pressure difference that generates the increase in circulation and thus lift.[13] This explanation is in agreement with Liebeck's hypothesis. All of the vortex structures involved are strictly with spanwise orientation, that is, parallel with the wing trailing edge. As already mentioned, a simple analysis relating the effectiveness of a Gurney flap in enhancing lift to the flap parameters was given by Gai and Palfrey.[17] Among the other researchers whose works have been reviewed and discussed earlier, there have been no attempts to explain the Gurney flap flow mechanism.

Before proceeding further, it is valuable to review Helmholtz's vortex laws at this point. A brief statement of these laws is given next. A somewhat detailed review of the discussions pertaining to these laws by the authors of about two dozen major textbooks on aerodynamics and fluid mechanics has been included in the appendix.

Theoretical Background: Vortex Theorems of Helmholtz

Almost a century and a half ago, Helmholtz[19] stated his celebrated vortex theorems, which still today, according to many, represent some of the most fundamental laws of aerodynamics. For inviscid flow of a barotropic fluid subject only to potential forces, his vortex laws can be stated as follows:

1) The strength of a vortex filament, or a vortex tube, remains constant along its length and is independent of time.

2) A vortex filament, or a vortex tube, cannot start or end in a fluid; it must start and end at a boundary, form a closed path, or extend to infinity.

3) The fluid particles that form a vortex filament, or a vortex tube, continue to form a vortex filament, or a vortex tube, and the strength of the vortex remains constant as it moves about.

Helmholtz's famous paper has been often revisited by numerous researchers in the field of theoretical aerodynamics (for example, see Refs. 20–42). To provide a historic context within which to assess the importance of these theorems, brief accounts of the treatises of Helmholtz's vortex laws by the authors of about two dozen well-recognized books on aerodynamics and fluid mechanics have been briefly reviewed and presented in the appendix to this paper.

Wing-Gurney Flap Flow Mechanism: Expanded Explanation

The present author is of the opinion that the explanation proposed by Liebeck, although valid, is not quite complete because the additional vortices caused by the presence of the Gurney flap, which have already been discussed, would have to comply with the vortex laws. Therefore, it is suggested that, according to the Helmholtz vortex theorems, the additional vortices as a result of the addition of the Gurney flap will not stop at the edges of the Gurney flap but rather turn and continue downstream in loops in a manner equivalent to that in which the classical wing horseshoe model is formed. Therefore, it appears that three additional trailing vortices will emanate at each wing tip, assuming that the Gurney flap is full span. The upper vortex behind the Gurney flap will be of the same sign as the wing trailing vortices, thus adding to the downwash. It is this vortex component that brings about the increased bound vortex circulation and thus additional lift. The lower vortex behind the Gurney flap and the one in front of it will be of the opposite sign relative to the wing trailing vortices, and it is quite possible that these vortices are responsible for the somewhat disrupted, or weaker, near-field vortex rollup in the presence of a Gurney flap, as it has been reported in Refs. 1 and 43.

Rationale for Present Study

For about a year now the author has been looking into the possibility to use Gurney flap as a trailing-wake vortex attenuation device. Several flap configurations have been investigated, and some promising results have been found.[1,43] It appears that Gurney flap affects at least the near-field wing vortex wake; marked differences have been observed in the wake vortex rollup patterns from a Gurney flap equipped vs a "clean" wing. The motivation for this study was of intuitive nature at first: knowing the history of the Gurney flap as a lift-, as well as drag-altering, device and the close relationship between a wing's lift and its vortex structure, on one hand, as well as the three-dimensional nature of the flow at a wing's trailing edge; on the other, it appeared that the presence of a Gurney flap there should have some impact on the ensuing vortex roll up.[1] When this proved to be the case, the argument was taken one step forward: if the wing trailing vortex filaments roll up differently when a Gurney flap is present, that situation is tantamount to having some additional *streamwise* (not just the spanwise vortices that are responsible for the increased circulation) vortex filaments springing from the wing trailing edge at the flap ends and propagating downstream in the vicinity of the wing trailing vortices, thus affecting their disposition. To effectively explain the existence of these additional trailing vortices, the Helmholtz vortex law, the one which states that a vortex filament cannot end in the fluid but must continue to infinity, needs to be invoked and shown to be true. When this is accomplished, the missing link appears to be found: the additional, Gurney-flap-generated bound vortex, because it cannot end in the fluid, bends at the flap edges and continues downstream as two free vortices. These flap trailing vortices, in turn, interact with the wing trailing vortices.

To examine the plausibility of this presumed scenario, an experimental study has been conducted with the objective to attempt to clearly identify additional trailing vortices in the case when a Gurney flap is installed. To accomplish this, it was decided to test a clean wing and three flapped-wing configurations. The three Gurney flap configurations would include one full-span flap, in which case the clean wing trailing ("tip") vortices should be augmented with the effect spreading along the whole wing span; one part-span Gurney flap centrally mounted, in which case two new trailing vortices originating one at each flap end and corotating with the wing tip vortices; and lastly two part-span flaps mounted from the tips inboard, in which case not only the tip vortices should become stronger but also a pair

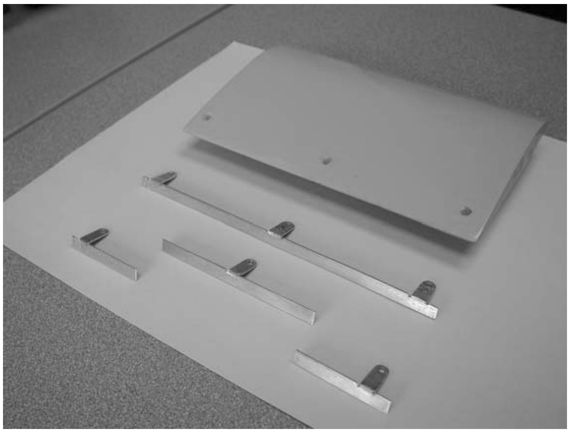

Fig. 2 Models of wing and Gurney flaps.

Fig. 3 Simple vorticity meter used in the study.

of counter-rotating vortices corresponding to the inner flap ends should be generated. If shown true, these three outcomes would undoubtedly prove that the installation of a Gurney flap results in additional trailing vortices, and thus the original flow hypothesis should be extended to include these streamwise vortices.

Experimental Methods

The wing model used in this study consisted of a rectangular wing having a NACA 4412 airfoil, a chord of 99.6 mm, and a span of 161.5 mm, thus having an aspect ratio of 1.62. The experiments have been conducted in the open-circuit wind tunnel at Minnesota State University, capable of producing wind speeds of up to 45.7 m/s. The test-section size is 305 × 305 mm. A detailed description of the tunnel and its instrumentation can be found in Ref. 44. The tests in this study have been performed at a Reynolds number based on the wing chord of about 0.225×10^6.

Three different Gurney flap models have been used. They all had a height of $0.06c$ and have been constructed from 0.8-mm-thick aluminum sheet including the attachment brackets. The first model represented a full-span Gurney flap, having three brackets. The second model consisted of a $0.5b$ flap having one bracket for attachment at the wing centerline. The third model comprised two $0.25b$ segments, each having one bracket for attachment from the respective wing tip to the point at $0.25b$ on each semispan. Figure 2 shows the wing and the three Gurney flap models. When installed, no leakage occurred between the wing and the Gurney flap. The four configurations investigated included 1) baseline wing (no flap), 2) wing with the full-span Gurney flap, 3) wing with $0.5b$ Gurney flap (inner), and 4) wing with $0.5b$ Gurney flap (outer).

To facilitate observation of the roll up of various vortices in the wing wake, the wing has been instrumented with 11 tufts of thin polyester thread. The tufts had a length of 242 mm ($1.5b$) each and were of different colors. They were uniformly spaced, 8 mm ($0.1s$) apart, across the port semispan. Good quality photographs of the ensuing vortex roll ups have been taken using a digital camera. (The wing without a flap has been referred to as the "baseline wing" rather then a clean wing because it had the tufts installed.)

A simple four-vane "vorticity meter," having a diameter of 30 mm, has been constructed and used in this study to measure the rotation in various vortices. It is shown in Fig. 3. It has been used in combination with a Shimpo digital stroboscope, Model DT-301. A 900-cell tuft grid has also been used for locating and observing vortex structures at various streamwise locations behind the wing trailing edge. It has a cell size of 9.5 mm, square. The tuft grid is shown in Fig. 4.

All of the models have been tested at $\alpha = 15$ deg. This condition has been selected as being representative of takeoff and landing conditions; the wing stalled at between 18 and 19 deg. After the tunnel has run at the nominal speed for a short period of time, it was brought to its minimal speed of approximately 4 m/s. Next, the vortex roll-up pattern was observed; the starting point of each vortex and the number of tufts entrained in it were recorded. Photographs of the flow visualization tufts were taken. Then the vorticity meter was immersed into each vortex at a constant distance from the trailing edge and the angular velocity, revolutions per minute, measured using the stroboscope. Before each run, the tufts were carefully untangled and straightened.

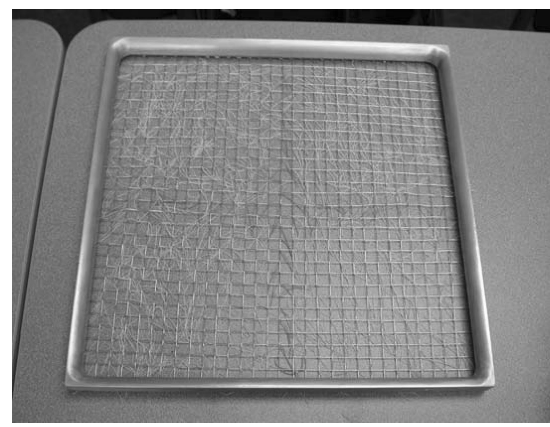

Fig. 4 Tuft grid with 900 cells.

The following are estimates of the experimental uncertainties. The angle of attack of the wing is determined to within ±0.25 deg. All lengths are reliable to within 0.5 mm. The dynamic pressure is accurate to within ±0.1 kPa. The rotational speed uncertainty is estimated at ±25 rpm.

Discussion of Results

Observed Vortex Structures

Baseline Wing

The flow-visualization tufts underwent what can be termed the classical pattern illustrated here in Fig. 5. Several runs were completed, and the level of repeatability of these patterns was found to be high. The largest concentration of vorticity in the wing-tip region causes the tufts in that area to roll into a strong vortex rope. Occasionally, a pair of inboard tufts would cross each other. No other vortical structure was formed.

Full-Span Gurney Flap

Figure 6 shows this configuration prior to its being installed into the tunnel. The experiment was repeated, and the resulting vortex structure is shown in Fig. 7. The trailing vortex still has the classical form as the one created by the baseline wing. The increased bound vortex strength over the same wing span produced stronger local vorticities shed into the wake. As a result, two inboard tufts, the ones located at $0.2s$ and $0.3s$, crossed one another. However, there are no organized vortex structures between the tip vortex and the wing centerline. Although the strength of the tip vortex has been increased by the addition of the Gurney flap, the presence of the flap appears to hinder the usual spanwise flows at the trailing edge and thus affects

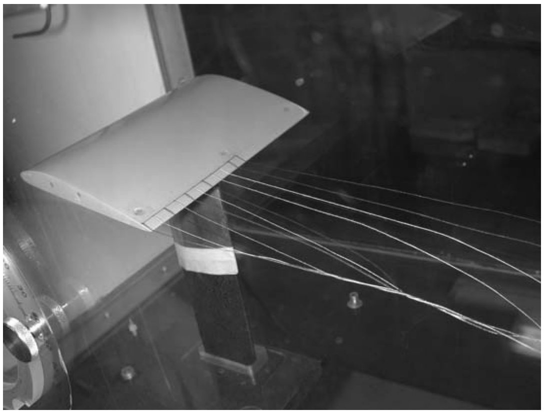

Fig. 5 Trailing vortex wake of baseline wing.

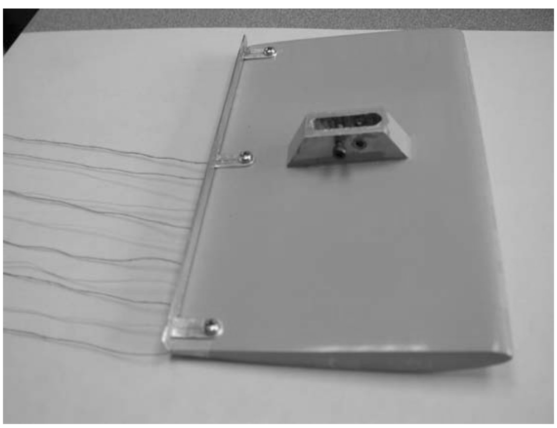

Fig. 6 Wing with full-span Gurney flap.

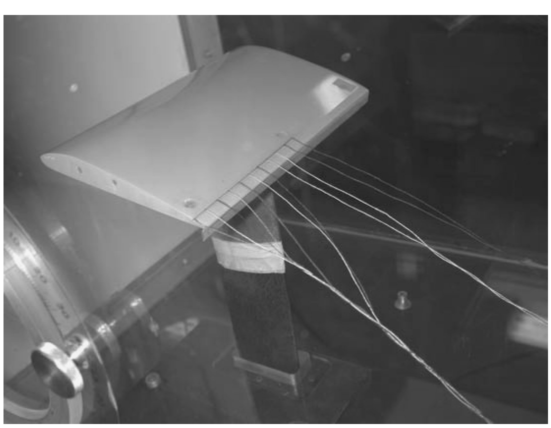

Fig. 7 Trailing vortex wake of wing with full-span Gurney flap.

the vortex roll-up process, at least in the near wake. These effects have been investigated separately and reported recently.[1,43]

Half-Span Inboard Gurney Flap

A very strong midspan vortex, corresponding to the outer edge of the flap, was observed. As expected, the sense of rotation was the same as that of the wing-tip vortex. As a result, a distinct inner vortex structure was formed comprising four tufts located around the midspan point. At the same time, the main wing-tip vortex structure consisted of five tufts. Figure 8 depicts this situation quite well. It was possible to follow the flap vortex using the vorticity meter up to $1.75b$ downstream of the wing. As the distance from the wing increased, this vortex moved downward and outboard—again as expected because of the strong interaction between it and the main wing-tip vortex. The main tip vortex was still very strong at $x = 2b$.

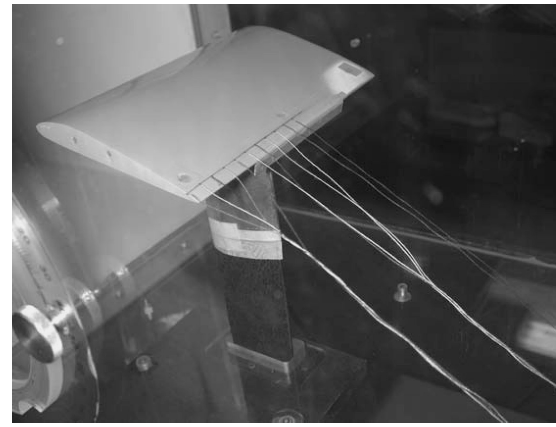

Fig. 8 Trailing vortex wake of wing with inner 0.5-span Gurney flap, $V = V$min.

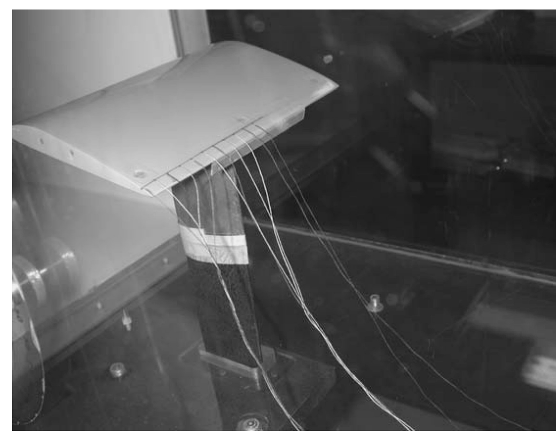

Fig. 9 Trailing vortex wake of wing with inner 0.5-span Gurney flap, $V = 0$.

Figure 9 shows a view at these two vortices after the tunnel had been stopped. One can clearly see a new vortex structure originating at the edge of the inner flap as would be expected in accordance with the Helmholtz vortex law.

Two Outer-Quarter-Span Gurney Flaps

The vortex roll up is shown in Fig. 10. Two distinctive vortices, one at the inner edge of each segment, were observed. These vortices had the sense of rotation opposite to the main wing-tip vortices, as expected. The vortices started at $y/s = 0.5$ and slightly below the trailing edge. I was able to positively identify and track the Gurney flap vortex up to $1.5b$ downstream from the trailing edge. At the same time, the main vortex was very strong at $2b$. As they moved downward and towards the wing tips, under the action of the wing-tip vortices, they gradually merged with the latter, as can be seen from this figure.

These tuft flow-visualization results confirmed the existence of additional streamwise trailing vortices in all cases, and the experimental results have been along the lines as discussed in a preceding section. These results indicated that indeed the presence of the Gurney flaps would cause additional streamwise vortices to be formed.

Throughout this study the tuft grid already described has been used extensively to aid in locating vortical structures. The grid has been placed at various downstream distances from the wing trailing edge, from $0.1b$ to $2.0b$. When used at less that $1.5b$, the vortex roll-up tufts were removed to avoid their tangling. Figure 11 shows the tuft grid flow visualization for the clean wing, $x/b = 1.0$.

Table 1 Measured angular velocities

Configuration	Tip vortex, rpm	Vortex @ $y/s = 0.5$, rpm
Baseline	3885, ccw	0
Full-span GF	5193, ccw	0
0.5-span GF, inner	4048, ccw	3272, ccw
0.5-span GF, outer	4630, ccw	2619, cw

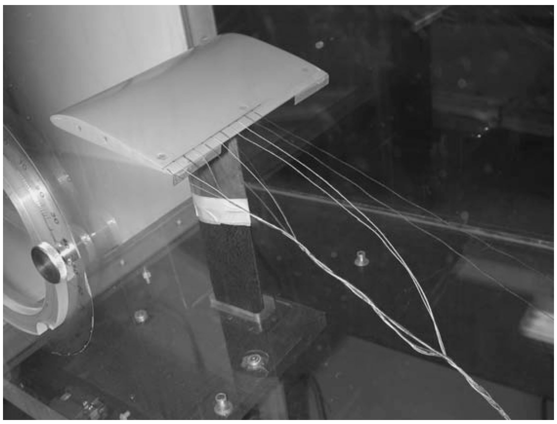

Fig. 10 Trailing vortex wake of wing with outer 0.5-span Gurney flap.

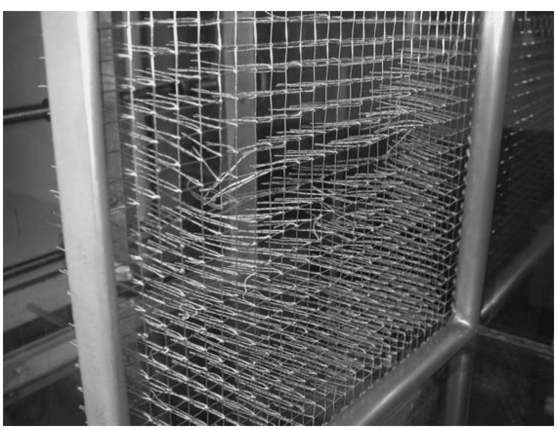

Fig. 11 Tuft grid flow visualization of baseline wing, *x*/*b* = 1.0.

Measured Rotational Velocities

Next the rotational velocities in the vortex regions were measured using the vorticity meter and the stroboscope already described. The measurements were made at the spanwise distances $y/s = 1.0$ and 0.5 and at streamwise locations corresponding to $x/b = 0.3$, or approximately 50 mm from the trailing edge. The results are given in Table 1 in revolutions per minute (rpm).

It can be seen from the data in Table 1 that the addition of the full-span Gurney flap (GF) caused an increase in the rotational speed in the tip vortex of 33.7% relative to the baseline value. Also, the half-span Gurney flaps led to a 4.4 and a 19.2% increase in this speed for the inner and outer flap, respectively. Most interestingly, the inner-flap vortex rotation amounted to 84.2% of the baseline wing speed, and the outer one to 67.4%, the latter being in the opposite direction.

Conclusions

An extension to the commonly accepted hypothesis for the Gurney flap flow mechanism has been proposed. In addition to the three spanwise vortices produced by the flap, which have been at the core of the explanation so far, an extension to this explanation has been put forward that, based on the Helmholtz vortex laws, additional streamwise vortices should exist, that is, these spanwise vortices ought to continue and bend at the tips of the flap and then extend downstream into the wing wake in the form of trailing vortices. To situate this work in the proper context, rather extensive reviews of both the relevant research work on Gurney flaps and of a large body of the widely accepted literature pertaining to the Helmholtz vortex theorems have been included. A series of wind-tunnel experiments have been conducted with the objective to demonstrate the existence of the additional vortices. Four configurations, including a baseline wing, one full- and two half-span Gurney flaps, have been tested in a low-speed wind tunnel. Tuft flow visualization has been used to capture the roll-up patterns in the near-field wing wake. This method has confirmed clearly that the proposed extension is valid. By measuring the rotational velocities in the tip- and midspan vortices, it has been further corroborated that the streamwise vortices indeed exist as suggested at the outset of this study.

Appendix: Helmhotz's Vortex Laws

In his classic paper published in 1858, the German scientist Herman von Helmholtz presented his famous vortex theorems,[19] which, even today, represent some of the most essential laws of aerodynamics. Next, the discussions of these laws by the authors of about two dozen prominent books on aerodynamics and fluid mechanics are reviewed in a chronological order.

Stalker[20] stated that the following laws for vortices in a perfect fluid have been deduced by Helmholtz:

1) The strength of a vortex is constant throughout its length, and for all time.

2) The particles of fluid, which at any time belong to a vortex, always remain in it.

Thus a vortex cannot begin or end in the fluid. It must either begin and end at a boundary, or extend to infinity, or return on itself, as does a smoke ring, for example.

Based on the results obtained by Helmholtz and Kelvin, Lamb points out the same important consequence of the preceding theorems: any vortex lines that exist must either form closed curves or else traverse the fluid, beginning and ending on its boundaries.[21]

Prandtl and Tietjens[22] summarize Helmholtz's vortex theorems as follows. For an inviscid fluid, subject to potential forces only, in addition to the two theorems already stated, the following theorem is also true: The product of the cross-sectional area and the angular velocity of an infinitely thin vortex filament is constant over the whole length of the filament as it moves. Thus, vortex filaments must be either closed or end on the boundaries of the fluid.[22] In his renowned book, von Mises states the following theorem: in a steady two-dimensional flow of an incompressible fluid, the rotation of any fluid element maintains its value as this element moves along.[23] He further points out that this result covers a very particular case of the famous theorem from Helmholtz. Thwaites asserts that, for inviscid flow, the main theorems concerning vortex lines were given by Helmholtz (1858) and Kelvin (1869). The bound vortex lines representing solid surfaces are hypothetical so that they are regarded as capable of sustaining a pressure and as not moving with the fluid.[24]

If, according to Prandtl, in studying the rotational motion of a homogeneous frictionless fluid, one begins with Thomson's theorem on the constancy of the circulation along a fluid line and the geometrical properties of the vector representing the rotation, then one can deduce theorems establishing important geometrical and mechanical relationships.[25] These are the same theorems as the ones that Helmholtz discovered by starting from electrodynamical considerations. Prandtl further points out that the results which follow from these theorems simplify if the region in which the fluid is in rotation has the form of a thread while the rest of the fluid is rotation free. This leads to the concept of a vortex filament. Then the following theorem is deduced: *a vortex filament cannot end at any point within the fluid, and the circulation about it is everywhere the same*. To this purely geometrical statement, we can add the dynamical statement following from Thomson's theorem, that *the circulation about the vortex filament remains unaltered as time goes on*.[25] An example is provided by the bound vortex representing a finite-span wing: when trailing vortices are added, the resulting system satisfies Helmholtz's theorem. Prandtl further points out that the theorem that a vortex filament can never end in a fluid and must have the same strength at every point is purely kinematic and therefore holds for a combination of free and bound vortices just as it does for free vortices.[25]

Robertson[26] explains that the vortex theorems of Helmholtz essentially represent a detailing of the Kelvin circulation theorem. It is recalled that Kelvin's theorem, also known as Thomson's theorem, concerns the circulation around a closed path, but because this is merely the strength, or moment of all vortices enclosed by the path, the theorem also applies to the vortex strength. As stated by Robertson, the vortex theorems for irrotational fluid motion are as follows:

1) A vortex line or filament can neither begin nor end in the fluid; hence, it appears as a closed loop or ends on a boundary.

2) Vortex tubes always contain the same fluid particles and vice versa—the vorticity is "frozen" into the material.

3) In a frictionless fluid the strength of any vortex filament—its moment—is constant with time.[26]

Ashley and Landahl[27] put forward that, in connection with the study of wing wakes, separation, and other related phenomena, it is helpful to study the vorticity ζ. However, because ζ is the curl of another vector, this field has certain unique properties. Two of these properties are identified by the first two vortex theorems of Helmholtz, which the authors state as follows:

1) The circulation around a given vortex tube—the "strength" of the vortex—is the same everywhere along its length.

2) A vortex tube can never end in the fluid, but must close onto itself, end at a boundary, or go to infinity. Examples of the three kinds of behavior are a smoke ring, a vortex bound to a two-dimensional airfoil, and the downstream ends of the horseshoe vortices representing a three-dimensional wing, respectively. This second theorem can be readily deduced from the continuity of circulation asserted by the first theorem. One simply notices that assuming an end for a vortex tube leads to a situation where the circulation is changing from one section to another along its length, which would violate the first theorem.[27]

Tokaty considers the flow for which $Curl\boldsymbol{\nu}$ is not equal to zero everywhere, that is, there exists in the flow a string of rotating elements, a vortex line.[28] Then, according to Helmholtz, these lines, which are the axes of rotation, have to either be closed curves, or they begin and end on the boundaries of the fluid or on the points in regions of infinite vorticity. If the flow is potential, that is, vortex free, and if the velocity potential $\varphi = \varphi(x, y)$ is a single-valued function of the coordinates (x, y), then $\Gamma = 0$ over any curve within the flow. Tokaty declares that Helmholtz proved that the strength of the vortex is constant throughout its entire length, that is, $\omega A =$ constant everywhere. This is known as Helmholtz's second law, the first being the Cauchy–Helmholtz law.[28]

According to Milne-Thompson, the product of the magnitude of the vorticity and the cross-sectional area, which the author calls *intensity*, is constant along a vortex filament.[29] The author refers to this statement as Helmholtz's second theorem. Then it follows from this theorem that a vortex filament cannot terminate in the interior of the fluid because, if it did, the cross-sectional area would have to be zero and therefore the vorticity would have to be infinite. The same argument has been made by Ashley and Landahl.[27] Thus a vortex filament must either be closed, for example a vortex ring, or it must terminate on the boundary. As an extension of the theorem, a vortex tube can be regarded as a bundle of vortex filaments, so that the term *intensity of a vortex tube* can be used to depict the sum of the intensities of the filaments that make up the tube. Then it follows that the intensity of a vortex tube remains constant along the tube. It is not necessary to assume steady motion. For completeness, two more Helmholtz's theorems for vortex tubes are stated:

3) The fluid that forms a vortex tube continues to form a vortex tube.

4) The intensity of a vortex tube remains constant as the tube moves about.[29]

White discusses some special cases of the Helmholtz equation of hydrodynamics in regard to its viscous-diffusion term.[30] For the ideal fluid, the equation leads to Helmholtz's theorem that the strength of a vortex remains constant as well as to Lagrange's theorem that vorticity is equal to zero at all time if it is equal zero everywhere at time $t = 0$. Still a third result of neglecting the viscous term is Kelvin's theorem that the circulation about any closed path moving with the fluid is constant.[30] Lugt points out that Helmholtz derived the famous vorticity theorems for the vorticity field of an inviscid fluid.[31] These theorems say, in essence, that vorticity in an inviscid fluid can neither be generated nor destroyed. Eleven years later, Lord Kelvin formulated another important version of Helmholtz's vorticity laws in the form of the circulation theorem named after him. These discoveries laid the basis for a modern vortex theory. The first theorem says that the strength of vorticity in a vorticity tube is the same in all cross sections. Furthermore, it states that a vorticity tube must be closed or must end at a boundary. Helmholtz's second theorem is valid only for ideal flows of incompressible fluids. It says that vorticity in such a flow can neither be generated nor destroyed because during the movement of vorticity the fluid particles cannot leave the vorticity line on which they are positioned.[31]

When discussing the vortex system of a thin planar wing of finite span, Moran explains that the equality of the strengths of the two tip vortices to those of the bound and starting vortices can be related to the one of Helmholtz's vortex theorems which states that a vortex cannot end in the fluid.[32] Namely, if there exists a vortex bound to the wing, it must continue at the wing tips back into the wake in the form of trailing vortices. Shevell explains that some basic properties of vortices are fundamental to the study of three-dimensional flow.[33] He continues by stating that these vortex laws, first given by Helmholtz, are as follows:

1) A vortex filament cannot end in the fluid. It must extend to infinity or form a closed path.

2) The strength of a vortex filament is constant along its length.

3) Vortices in a fluid always remain attached to the same particles of fluid.

An additional important theorem is the one attributed to Thomson, or Kelvin, which states that the circulation around any path that always encloses the same fluid particles is independent of time. He illuminates that these theorems are applicable to a perfect fluid, but are good approximations to viscous fluid flows in regions where viscosity can be neglected. They are based on the concept that no tangential forces can be applied to a fluid particle and that the angular velocity of the particles must therefore remain constant.[33]

Katz and Plotkin[34] state that, based on results similar to those pertaining to Kelvin's theorem, Helmholtz developed his vortex theorems for inviscid flows, which can be summarized as follows:

1) The strength of a vortex filament is constant along its length.

2) A vortex filament cannot start or end in a fluid. (It must form a closed path or extend to infinity).

3) The fluid that forms a vortex tube continues to form a vortex tube, and the strength of the vortex tube remains constant as the tube moves about. Therefore vortex elements, such as vortex lines, vortex tubes, vortex surfaces, etc., will remain vortex elements with time.[34]

In his prominent textbook, McCormick asserts that, according to the Helmholtz theorem regarding vortex continuity, a vortex line or filament can neither begin nor end in a fluid; hence, it appears as a closed loop, ends on a boundary, or extends to infinity.[35]

When discussing Helmholtz's laws for inviscid flow, Panton[36] points out that, if viscous diffusion is not significant and body forces have a potential, the behavior of vorticity follows three laws from Helmholtz. He explains that in many instances a fluid acquires vorticity by viscous action and then the subsequent motion is inviscid; the viscous forces are negligible. He states Helmholtz's laws in the following way:

1) No element of fluid, which was not originally in rotation, is made to rotate.

2) The elements that at any time belong to one vortex line, however they may be translated, remain on one vortex line.

3) The product of the section and the angular velocity of an infinitely thin vortex filament is constant throughout its whole length, and retains the same value during all displacements of the filament. Hence, vortex filaments must be closed curves or must have their ends in the bounding surface of the fluid. This essentially means that $\Gamma =$ constant applies for all time to a material vortex tube moving with the fluid.[36]

In their famous textbook, Kuethe and Chow[37] present the statements and proofs for Helmholtz's vortex theorems. Only the statements will be repeated here. They are as follows:

1) The strength of a vortex filament is constant along its length.

2) A vortex filament cannot end in a fluid; it must extend to the boundaries of the fluid or form a closed path.

3) In the absence of rotational external forces, a fluid that is initially irrotational remains irrotational.

Schetz and Fuhs[38] discuss the three Helmholtz vortex laws as the three basic laws for motion of a vortex filament in an otherwise irrotational fluid. The authors indicate that these laws—essentially corollaries to Kelvin's theorem—are as follows:

1) A vortex filament can neither begin nor end in the fluid; it must form a closed loop or else end on the fluid boundary.

2) Vortex tubes always contain the same fluid particles.

3) The strength Γ of a vortex line is constant. Hence, from Stokes' theorem the product of its vorticity and its cross-sectional area must be constant.[38]

When discussing kinematics of vorticity and circulation, Warsi starts by writing the vorticity fluxes through two arbitrarily chosen surfaces.[39] This leads him to conclude that the flux of vorticity through any section of the vortex tube must remain unchanged. Next, he shows that *the circulations about the curves* C_1 *and* C_2 *on a vortex tube at a given instant are the same. This result is known as the first theorem of Helmholtz*. Here C_1 and C_2 are the section curves of the vortex tube. He also makes an argument identical to that made by Ashley and Landahl[27] and by Milne-Thompson[29] that a vortex tube cannot end in the fluid. Cottet and Koumoutsakos[40] observe that the strength of the vortex tube is the same at all cross sections, which is Helmholtz's first vortex theorem. Also, in a circulation-preserving motion the vortex lines are material lines—Helmholtz's second theorem. A result of the first and second Helmholtz's vortex laws is the following important property of vortex lines and tubes: no matter how they develop, they must always form closed curves, or they must have their ends in the bounding surface of the fluid, that is, they cannot start or end in the fluid.[40]

In his well-known textbook on aerodynamics, Bertin states that vortex lines have an important role in the study of the flow around wings.[41] He summarizes the vortex theorems of Helmholtz as follows. For a barotropic, homogeneous, inviscid flow acted upon by conservative forces only, the following statements are true:

2) The circulation around a given vortex line is constant along its length.

3) A vortex filament cannot end in a fluid. It must form a closed path, end at a boundary, or go to infinity.

4) No fluid particle can have rotation, if it did not originally rotate.[41]

Kundu and Cohen discuss Helmholtz's vortex theorems for the conditions of inviscid flow, conservative body forces, barotropic fluid, and a nonrotating frame.[42] Then the theorems are as follows:

1) Vortex lines move with the fluid.

2) Strength of a vortex tube is constant along its length.

3) A vortex tube cannot end within the fluid. It must either end at a solid boundary or form a closed loop—"a vortex ring."

All of the works briefly discussed here assert the property of vortex filaments or vortex tubes that they cannot start or end in a fluid. The fact that the different authors can refer to this property as one or another among the Helmholtz's vortex laws is irrelevant; what is important is the assertion that it is a principal property of vortex flows in inviscid fluids, and a good approximation in real fluid flows.

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